Contents

0	About this book					
1 Mercury By Example						
	1.1	Hello, World!	5			
	1.2	The Fibonacci numbers	10			
	1.3	Simple input	14			
	1.4	rot13	19			
	1.5	Cryptarithms	23			
2	The	Mercury type system	27			
	2.1	The primitive types: int. float. string. and char	28			
		\mathbf{r} $(0, \mathbf{r})$				
	2.2	Tuples	30			
	2.2 2.3	Tuples Lists	30 31			
	2.22.32.4	Tuples	30 31 31			
	 2.2 2.3 2.4 2.5 	Tuples	30 31 31 34			
	 2.2 2.3 2.4 2.5 2.6 	Tuples	 30 31 31 34 36 			

	2.8	Higher order types	38
	2.9	univ, the universal type	40
	2.10	Useful types defined in the Mercury standard library	41
	2.11	More advanced types	42
3	The	Mercury mode system	43
	3.1	Predicates and procedures	43
	3.2	The determinism categories	45
	3.3	Determinism	47
	3.4	Procedures and code reordering	51
	3.5	Insts and subtypes	53
	3.6	Uniqueness	55
	3.7	Higher-order modes	57
	3.8	Committed-choice nondeterminism	57

Chapter 0

About this book

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Disclaimer.

This was a work in progress. The author has ceased active involvement with Mercury and there are (currently) no plans to continue work on it.

What is this book for?

A basis for courses teaching or using Mercury. A tutorial for experienced programmers wishing to learn Mercury.

What version of Mercury is used in the book?

Any recent release-of-the-day (ROTD) compiler will do. The text as of today (10 April 2005) will work fine for Mercury release 0.12.

Who is it aimed at?

Students who have completed at least one year of a computer science degree, been exposed to a strongly, statically typed functional programming language (e.g., ML, Haskell, Miranda), and have some reasonable programming experience.

What is this book about?

This book is a tutorial on programming in Mercury. The book should provide the reader with enough information to become a competent Mercury programmer and knowledge sufficient to understand the Mercury Reference Manual, the Mercury Library Reference Manual and the Mercury Users' Guide. The book will describe key features of Mercury in detail, including the type system, the mode and determinism systems, and programming with nondeterminism.

What is this book not about?

This book will not teach you how to program in general, nor will it teach you a great deal of theory. It will not teach you how type inference works; it will (probably) not teach you about Herbrand universes; it will not include an extended treatise on IO in declarative programming languages; it will not (in the first place) cover black-belt Mercury programming techniques (e.g. dynamic type casts for non-ground types).

Where is the introduction?

I plan to write the introduction last. It will include a brief description of "what Mercury is all about" and an explanation of what the book is trying to do.

Chapter 1

Mercury By Example

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This chapter aims to convey through examples a basic feeling for how Mercury works. Because this is the first chapter, we may gloss over some fine detail and allow ourselves a certain latitude in precision, but this will not worry us as we will correct these deficiencies in later chapters. The approach taken here is to start by presenting the "obvious" solution to a problem and then introduce features of Mercury that allow for more elegant or efficient programs.

1.1 Hello, World!

It is slightly unfortunate that the "Hello, World!" program introduces no less than three advanced Mercury concepts, but since tradition dictates that tutorial texts start with "Hello, World!" we'll just have to jump straight in with the knowledge that things will get easier thereafter.

We'll start by presenting the complete program which we'll assume we've typed into a file called hello.m:

```
:- module hello.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is det.
:- implementation.
```

```
main(IOState_in, IOState_out) :-
    io.write_string("Hello, World!\n", IOState_in, IOState_out).
```

Let's go through this line by line.

:- module hello.

Every module must start with a declaration like this giving the name of the module; the Mercury compiler will expect a module called hello in a file called hello.m.

:- interface.

An interface declaration introduces the part of module describing the things we are going to export (i.e., allow users of the module to see).

```
:- import_module io.
```

This import_module declaration says that we need to use some of the names exported by the io module, which is part of the standard Mercury library.

:- pred main(io::di, io::uo) is det.

This says that we are going to define a *predicate* called **main** with two arguments of type **io** (which happens to be defined in the **io** module), the first of which is a *destructive input* argument, and the second of which is a *unique output* argument; the **is det** part tells us that **main** is a *deterministic* predicate — that is, **main** always succeeds and will always compute the same output given the same input (we will see later that some Mercury predicates can fail and some can have more than one solution).

Every Mercury program needs to export a predicate called **main**, which the compiler takes as the starting point for the Mercury program as a whole.

:- implementation.

Everything after an implementation declaration is considered private implementation detail not visible to other users of the module.

```
main(IOState_in, IOState_out) :-
    io.write_string("Hello, World!\n", IOState_in, IOState_out).
```

Finally, we have a *clause* defining main. A clause comprises a *head* and *body* separated by a :- symbol. The *head* of this clause tells us that this is a definition for main and names its two arguments IOState_in and IOState_out. The body of the clause, which is executed when main is called, consists of a single *goal* calling io.write_string (i.e., the write_string predicate defined in the io module) with a message (Mercury interprets the \n sequence in the string argument as a literal newline character) and the two IOState arguments.

Program variables in Mercury always start with a capital letter or underscore, while names of predicates, types, modules and so forth do not.

We can compile and run hello.m as follows (\$ indicates the command line prompt):

```
$ mmc --make hello
Making Mercury/int3s/hello.int3
Making Mercury/cs/hello.c
Making Mercury/os/hello.o
Making hello
$ ./hello
Hello, World!
```

Et voilà! (By default, mmc --make will construct a local Mercury directory, if necessary, to hold intermediate files generated during compilation.)

At this point the reader is probably wondering about the meaning of the io type arguments IOState_in and IOState_out, and the strange di and uo argument modes. The short answer is that every predicate that performs I/O has to have an io type input argument describing the state of the world at the time the predicate is called and an io type output argument describing the state of the world after the call. This is how Mercury allows programs to communicate with the outside world without hidden side effects which would compromise its mathematical integrity.

Because it doesn't make much sense to try reusing an old io state — you can't go back in time — io states are *unique*. This is where the di and uo argument modes come in: they are just like the ordinary in and out argument modes we'll meet in the next section, except that they also specify

uniqueness. The Mercury compiler will not allow programs to copy unique values or reuse dead ones: there is only ever *one* "live" I/O state at a time in a running Mercury program. This property guarantees that I/O operations occur in the intended order.

So what if we want to do more than one I/O operation? In this case we have to give names to each of the intermediate io states:

```
main(IOState_in, IOState_out) :-
    io.write_string("Hello, ", IOState_in, IOState_1),
    io.write_string("World!", IOState_1, IOState_2),
    io.nl(IOState_2, IOState_out).
```

The first call to io.write_string takes IOState_in as an input, destroys it in the process of writing its string argument, and produces IOState_1 as its result. Then the second call to io.write_string destroys IOState_1 and produces IOState_2. Finally, io.nl (which just writes out a newline), destroys IOState_2 and returns IOState_out, which is the result of the call to main.

Naming all these intermediate states quickly becomes tedious, so Mercury provides us with syntactic sugar in the form of *state variables*:

```
main(!IO) :-
    io.write_string("Hello, ", !IO),
    io.write_string("World!", !IO),
    io.nl(!IO).
```

This code is transformed by the compiler into something equivalent to the preceding example: each occurrence of the **!IO** state variable actually stands for two normal variables, which are given intermediate names in the "obvious" way (the full details of the state variable transformation can be found in the Mercury Reference Manual). Note that there is no special significance in the name **IO**, which we merely use by convention.

Points to remember

- A module starts with a module declaration and is followed by an interface section and an implementation section.
- The interface section declares the things that are exported by the module.

1.1. HELLO, WORLD!

- All declarations start with a :- symbol.
- Declarations and clauses always end with a full stop.
- We have to import a module before we can use things exported by that module.
- Every Mercury program must export a predicate called main.
- Predicates are defined by clauses, which go in the implementation section.
- Variable names start with a capital letter; names of modules, types, predicates and so forth start with a lower-case letter or use symbols (e.g., the int module defines + to mean integer addition).
- Every predicate that performs I/O must have an io:::di argument and an io:::uo argument and must be deterministic.
- We use state variables to avoid having to explicitly name every io state.

1.2 The Fibonacci numbers

Another great computer science tradition is defining a function to calculate numbers in the Fibonacci series: 1 1 2 3 5 8 13 21 34... where, apart from the leading 1s, each number is the sum of the preceding two.

As before, we'll start by showing a complete program and then look at the interesting parts in more detail.

```
:- module fib.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is det.
:- implementation.
:- import_module int.
:- pred fib(int::in, int::out) is det.
fib(N, X) :-
    ( if
             N =< 2
       then X = 1
       else fib(N - 1, A), fib(N - 2, B), X = A + B
    ).
main(!IO) :-
   fib(17, X),
    io.write_string("fib(17, ", !IO),
    io.write_int(X, !IO),
    io.write_string(")\n", !IO).
Building and running fib.m, we find that...
```

```
$ mmc --make fib
Making Mercury/int3s/fib.int3
Making Mercury/cs/fib.c
Making Mercury/os/fib.o
Making fib
$ ./fib
fib(17, 1597)
```

The first thing to note is the import_module declaration at the start of the implementation section. We need to import the int module from the Mercury standard library because it defines all the operations on integers, such as addition and comparison. We import it in the implementation section rather than the interface section because that's the only place we refer to names defined in the int module.

Next the declaration

```
:- pred fib(int::in, int::out) is det.
```

says that we are going to define a predicate fib taking two int arguments, an input and an output, which always succeeds and always computes the same output given the same input.

```
fib(N, X) :-
  ( if N =< 2
    then X = 1
    else fib(N - 1, A), fib(N - 2, B), X = A + B
).</pre>
```

The body of the fib definition uses an if-then-else goal to decide what to do; the else part is not optional; and the whole thing appears in parentheses. The condition $N \ll 2$ succeeds if N is less than or equal to 2 and *fails* otherwise (we'll learn more about semidet predicates like =< in later examples). If $N \ll 2$ then the *unification* X = 1 is executed. Otherwise fib is called twice to compute the preceding two Fibonacci numbers in A and B, and X is unified with their sum (note that we don't need to declare A and B anywhere).

Finally we have

```
main(!IO) :-
   fib(17, X),
    io.write_string("fib(17, ", !IO),
    io.write_int(X, !IO),
    io.write_string(")\n", !IO).
```

which calls fib(17, X), unifying X with the result of computing the 17th Fibonacci number, then writes out the answer.

Now, just as N - 1 computes N minus 1 and A + B computes the sum of A and B, it is possible to define fib so that fib(N) computes the Nth Fibonacci number:

```
:- func fib(int) = int.
fib(N) = X :-
   ( if   N =< 2
      then   X = 1
      else   X = fib(N - 1) + fib(N - 2)
).</pre>
```

The func declaration introduces fib as a *function* with an int argument computing an int result. Mercury assumes that the input arguments to a function have mode in, the result has mode out, and that the function as a whole is det.

Functions are also defined using clauses, the only difference being that the head of a function clause takes the form FuncCall = Result.

Having redefined fib as a function, we also have to change how it is called in main:

```
main(!IO) :-
    io.write_string("fib(17) = ", !IO),
    io.write_int(fib(17), !IO),
    io.nl(!IO).
```

Here is one last refinement we might like to make to our definition of fib:

```
fib(N) = (if N = \langle 2 then 1 else fib(N - 1) + fib(N - 2)).
```

By using an if-then-else *expression* we can move the entire body into the head of the clause. Since the clause now has an empty body, Mercury requires that we omit the :- part.

It is worth noting that all three of our definitions are computationally identical and there is no difference in the code generated by the Mercury compiler. Whether to use a predicate or function is largely a matter of taste; a good rule of thumb is that if a predicate has a single output argument, and is deterministic, then it is probably better expressed as a function.

Points to remember

- Import modules in the implementation section only if they are only referred to in the implementation section.
- Non-unique arguments (typically non-io arguments) normally use the in and out argument modes.
- You can use functions to make your code more readable.
- if-then-else can be used as a goal and as an expression; it should appear in parentheses and the else part is mandatory.
- If a clause of a predicate or a function has an empty body (because all the computation is described in the head), then the :- must be omitted.

1.3 Simple input

We can extend our Fibonacci program to read in N rather than have it hard-coded by changing the definition of main:

```
:- import_module list, string.
main(!IO) :-
    io.read_line_as_string(Result, !IO),
    ( if
        Result = ok(String),
        string.to_int(string.strip(String), N)
        then
        io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO)
        else
        io.format("That isn't a number...\n", [], !IO)
).
```

The list and string standard library modules are imported in the implementation section because we use them in the definition of main (the io module has already been imported in the interface section).

main begins by calling io.read_line_as_string, which reads a whole line of input up to and including the next newline character. If all goes well then Result ends up unified to a value ok(String), where String is the string of characters read in and ok is a "tag" known as a *data constructor*. Other possibilities for Result are eof, indicating the end-of-file has been reached, and error(ErrorCode), indicating that something went wrong.

Then an if-then-else decides what to do. The if condition succeeds if Result is an ok value (unifying String with the argument) and if string.to_int(string.strip(String), N) succeeds. The string.strip function returns its argument minus any leading and trailing whitespace, including the terminating newline character, while the predicate string.to_int succeeds if its first argument is a string of decimal digits (unifying N with the corresponding number), and fails otherwise.

It's worth taking a slightly closer look at the unification in the if part of the if-then-else goal:

Result = ok(String)

We know that **Result** has a value at this point, so this kind of unification is known as a *deconstruction*: it only succeeds if the value in **Result** matches the pattern ok(*something*), in which case it unifies **String** with the *something*.

The then and else arms of the if-then-else goal call the io.format predicate, which is rather like C's printf function. The first argument is a format string (where %d indicates a decimal integer, %f indicates a floating point value, %s indicates a string, and %c indicates a character) and the second argument is a list of the corresponding values in [brackets], tagged with i, f, s or c for int, float, string or char values respectively. Thus in

```
io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO)
```

the int N is printed instead of the first %d in the format string and the int result of fib(N) is printed instead of the second %d. If N = 17 we'd expect this goal to output

fib(17) = 1597

Just as the Mercury compiler doesn't know how to do anything with integers unless you import the int module, it doesn't know anything about lists unless you import the list module. So if you want to use io.format then you also have to import the list and string modules.

Okay, so far so good. What if we want to input more than one number in a loop? That's easy to arrange:

```
main(!IO) :-
    io.read_line_as_string(Result, !IO),
    ( if
        Result = ok(String),
        string.to_int(string.strip(String), N)
        then
        io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO),
        main(!IO)
        else
        io.format("I didn't expect that...\n", [], !IO)
).
```

Now the **then** branch calls **main** recursively to read in another number. Being a declarative language, recursion is Mercury's only looping construct. However, as with any decent declarative language compiler worth it's salt, tail recursion like this is just as efficient as a **while** or **for** loop in any other language.

Firing up the compiler we get

```
$ mmc --make fib
Making Mercury/int3s/fib.int3
Making Mercury/cs/fib.c
Making Mercury/os/fib.o
Making fib
$ ./fib
| 10
fib(10) = 55
| 17
fib(17) = 1597
| 20
fib(20) = 6765
| ^D
I didn't expect that...
```

(The |s indicate input from the user and don't actually appear on the screen. ^D indicates the user typing Ctrl-D to close the input stream; Windows users should use Ctrl-Z.)

It would be good to handle the end-of-file condition more gracefully. The obvious way to do that is to add another case to the if-then-else:

```
main(!IO) :-
    io.read_line_as_string(Result, !IO),
    (    if
        Result = eof
        then
            io.format("bye bye...\n", [], !IO)
        else if
        Result = ok(String),
        string.to_int(string.strip(String), N)
        then
        io.format("fib(%d) = %d\n", [i(N), i(fib(N))], !IO),
        main(!IO)
```

```
else
    io.format("I didn't expect that...\n", [], !IO)
).
```

Observe the deconstruction test of Result with eof — the eof data constructor has no argument list and indeed it is a syntax error to write eof().

Before we leave our fib example, let us introduce Mercury's *switch* goals. A switch goal is rather like C's **switch** statement and consists of a set of alternatives testing a given variable against different possible values it might have. Here is **main** rewritten to use a switch goal:

```
main(!IO) :-
    io.read_line_as_string(Result, !IO),
    (
        Result = eof,
        io.format("bye bye...\n", [], !IO)
    ;
        Result = ok(String),
                 string.to_int(string.strip(String), N)
        (
          if
           then io.format("fib(%d) = %d n", [i(N), i(fib(N))], !IO)
           else io.format("that isn't a number\n", [], !IO)
        ),
        main(!IO)
    ;
        Result = error(ErrorCode),
        io.format("%s\n", [s(io.error_message(ErrorCode))], !IO)
    ).
```

A sequence of goals separated by semicolons is called a *disjunction* (the semicolon is usually pronounced "or"). If each *disjunct* deconstructs a particular variable against a set of mutually exclusive possibilities, then the disjunction as a whole is a switch. In general it is good style to use a switch rather than a sequence of *if-then-else* goals since then, in most cases, the Mercury compiler will tell you if you've forgotten a possibility or counted the same possibility twice.

Points to remember

• Some types (such as the result type of io.read_line_as_string) use different data constructors for different values. These values can be tested using deconstruction unifications.

- A data constructor with no arguments, such as **eof**, is not followed by an argument list.
- io.format is Mercury's version of C's printf. To use it you must import list and string as well as the io module.
- You should omit the parentheses around an if-then-else that immediately follows the else part of another if-then-else.
- A disjunction is a sequence of goals separated by semicolons.
- A switch is a disjunction where each disjunct tests a particular variable against a different possibility. Where applicable, switches are generally preferable to if-then-elses.

1.4. ROT13

1.4 rot13

Let's move on to a different example. This time we are going to implement the rot13 "encryption" algorithm, which works by rotating the Roman alphabet by 13 places — in other words, abcdefghijklmnopqrstuvwxyz in the input becomes nopqrstuvwxyzabcdefghijklm in the output. Decryption is simple: just use rot13 a second time! While rot13 has the cryptographic strength of damp tissue paper, it is sometimes useful for obscuring information in an e-mail that the recipient may not yet wish to know, such as who won the Grand Final.

Here's a first cut at a solution:

```
:- module rot13.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is det.
:- implementation.
:- import_module char, list, string.
main(!IO) :-
    io.read_char(Result, !IO),
    (
        Result = ok(Char),
        io.write_char(rot13(Char), !IO),
        main(!IO)
    ;
        Result = eof
    ;
        Result = error(ErrorCode),
        io.format("%s\n", [s(io.error_message(ErrorCode))], !IO)
    ).
:- func rot13(char) = char.
rot13(Char) = (
                  if
                            Char = 'a'
                                        then 'n'
                  else if Char = 'b'
                                        then 'o'
                   . . .
                                        then 'm'
                  else if Char = 'z'
                  else if Char = 'A'
                                        then 'N'
```

```
else if Char = 'B' then '0'
...
else if Char = 'Z' then 'M'
else Char
).
```

While this plainly works:

```
$ mmc --make rot13
Making Mercury/int3s/rot13.int3
Making Mercury/cs/rot13.c
Making Mercury/os/rot13.o
Making rot13
$ ./rot13
| Port Adelaide beat the Brisbane Lions 113 to 73 in the Grand Final.
Cbeg Nqrynvqr orng gur Oevfonar Yvbaf 113 gb 73 va gur Tenaq Svany.
| Cbeg Nqrynvqr orng gur Oevfonar Yvbaf 113 gb 73 va gur Tenaq Svany.
Port Adelaide beat the Brisbane Lions 113 to 73 in the Grand Final.
```

it's hardly going to win prizes for elegance or efficiency. A more experienced Mercury programmer might code rot13 like this:

```
:- func rot13(char) = char.
rot13(CharIn) = ( if rot13_2(CharIn, CharOut) then CharOut else CharIn ).
:- pred rot13_2(char::in, char::out) is semidet.
rot13_2('a', 'n').
rot13_2('b', 'o').
...
rot13_2('z', 'm').
rot13_2('A', 'N').
rot13_2('B', 'O').
...
rot13_2('Z', 'M').
```

There are three new things here: the **semidet** determinism category; clauses with the arguments already "filled in"; and using more than one clause to define a predicate.

1.4. ROT13

First off, the semidet determinism category means that $rot13_2$ will, for any given input, either fail or have a single solution for the output. Looking at the code we might guess (correctly) that $rot13_2('z', X)$ should succeed unifying X = 'm' (and never anything else), while $rot13_2('7', X)$ would fail.

Secondly, a clause like

rot13_2('a', 'n').

is just syntactic sugar for

rot13_2(V1, V2) :- V1 = 'a', V2 = 'n'.

Since we know from the pred declaration for rot13_2 that V1 is an input and V2 an output, the unification V1 = 'a' must be a deconstruction test and, if that succeeds, then the *construction* unification V2 = 'n' is carried out (a construction unification always succeeds because the "destination" variable, V2 in this case, does not have a value before this point).

Finally, a sequence of clauses is syntactic sugar for a single clause whose body is a disjunction. Hence our code is transformed by the compiler into this:

```
rot13_2(V1, V2) :- ( V1 = 'a', V2 = 'n'
; V1 = 'b', V2 = 'o'
....
; V1 = 'z', V2 = 'm'
; V1 = 'A', V2 = 'M'
; V1 = 'B', V2 = '0'
....
; V1 = 'Z', V2 = 'M' ).
```

The astute reader will immediately identify this as a switch on V1 because each disjunct tests the input V1 for a different possible value. One of the good things about switches is that the Mercury compiler will generate very efficient code for them, using a lookup-table or hash-table perhaps, which will certainly out-perform the long if-then-else chain in our first attempt.

(As an aside, the is semidet determinism declaration for rot13_2 tells the Mercury compiler that this predicate is expected to fail in some cases, so

it will not warn us about missing possible values for the first argument. In the fib program the switch had to be exhaustive, so it could not fail, because main was declared to be det. Had we missed a possible case out of that switch, the compiler would have reported the missing case as an error. Similarly, because rot13_2 cannot have more than one solution for any input, the compiler will report an error if we have duplicate clauses matching the same input. The compiler can't warn us about such problems if we use if-then-else chains. Try it and see what happens.)

Note that if we felt so inclined, we could make rot13_2 deterministic by including the translation of every possible character! Of course, there are many ways of coding rot13 and while our implementation may not be the most concise, it is quite efficient and very easy to understand. Either way, our aim here was to look more closely at the concept of semideterminism and introduce the technique of making code more readable by using multiple clauses to define a predicate or function.

Points to remember

- Literal character values in Mercury are normally enclosed in single quotes. Some characters which the Mercury parser would normally expect to see used as infix function symbols, such as + and *, also need to be enclosed in parentheses: ('+') and ('*'). Full details of how special characters should be written can be found in the Mercury Reference Manual.
- The semidet determinism category means that a predicate can have at most one *solution* for a given set of inputs (it fails if it has no solution for the given inputs).
- Semideterministic predicates therefore often appear in the conditions of if-then-else goals.
- "Filling in" the arguments of a clause is just shorthand for omitting the equivalent unification goals.
- The compiler views a predicate or function definition comprising several clauses as a single disjunction. Multiple clauses are often easier to read than a large disjunction.

1.5 Cryptarithms

In this example we introduce nondeterminism to demonstrate a little of what differentiates Mercury from conventional programming languages. A cryptarithm is an equation (usually just an addition) where the digits of each number have been replaced by letters, DOG + ANT = CAT for example; a solution is a mapping from letters to digits that satisfies the equation. Leading letters cannot stand for zero and each letter must stand for a distinct digit. Here's a Mercury program to solve this particular cryptarithm:

```
:- module crypt.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is cc_multi.
:- implementation.
:- import_module int, list, string.
main(!IO) :-
    io.format("DOG + ANT = CAT \setminus n", [], !IO),
    ( if
        Ds0 = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9], C0 = 0,
        pick(Ds0, G, Ds1),
        pick(Ds1, T, Ds2),
                            S1 = G+T+C0, T = S1 \mod 10,
                                                          C1 = S1/10,
        pick(Ds2, 0, Ds3),
                            S2 = 0+N+C1, A = S2 \mod 10, C2 = S2/10, A \ge 0,
        pick(Ds3, N, Ds4),
        pick(Ds4, D, Ds5),
                            S3 = D+A+C2, C = S3 \mod 10, C3 = S3/10, C \ge 0,
        pick(Ds5, A, Ds6),
        pick(Ds6, C, _),
        C3 = 0
      then
        DOG = 100 * D + 10 * 0 + G,
        ANT = 100 * A + 10 * N + T,
        CAT = 100 * C + 10 * A + T,
        io.format("%d + %d = %d\n", [i(DOG), i(ANT), i(CAT)], !IO)
      else
        io.format("has no solutions\n", [], !IO)
    ).
:- pred pick(list(int)::in, int::out, list(int)::out) is nondet.
```

pick([X | Xs], X, Xs).
pick([X | Xs], Y, [X | Zs]) :- pick(Xs, Y, Zs).

At this early stage we won't explain exactly how this program works. Instead, we will describe *what* it does and leave the technical explanation to later chapters.

The interesting part of this program is the condition of the if-then-else goal. The trick to understanding this code fragment is to consider it as a set of constraints on a solution rather than as a series of computations. (Concentrating on the "what" rather than the "how" is a hallmark of declarative programming.)

Ds0 is the list of digits from 0 to 9, Ds1 is Ds0 with the digit for G removed, Ds2 is Ds1 with the digit for T removed, Ds3 is Ds2 with the digit for O removed, and so forth. In this way we ensure that each of D, O, G, A, N, T and C are different.

The digits for each letter are selected nondeterministically using the pick predicate. The goal pick(Ds0, G, Ds1), for instance, picks a digit for G from Ds0 and leaves the remaining digits in Ds1. Since Ds0 contains ten members, there are ten possible solutions for G, and Ds1 will have nine members.

C1 is the carry from the units column, C2 is the carry from the tens column, C3 is the carry from the hundreds column; we introduce C0 as a "carry in" of zero to give the program a regular structure.

The goal S1 = G + T + C0, $T = S1 \mod 10$ provides the constraint for the units column. We do the same thing for the tens and hundreds columns.

The goals $A \ge 0$ and $D \ge 0$ ensure that we don't have any zeroes in the hundreds column ('\=' means "not equal to"), while C3 = 0 ensures that there is no carry left over from the hundreds column.

Eventually either a set of solutions to the pick goals will be found that is consistent with the other constraints or, if no such set exists, the condition of the if-then-else will fail.

At this point we can explain the cc_multi determinism category for main, which is different to det which we've used in all the preceding examples. cc_multi stands for *committed choice multideterministic*, which means that although this predicate may have multiple possible answers (there may be

multiple solutions to main's if-then-else condition) we will only be getting one of them! This extra constraint is necessary to allow a predicate with more than one possible solution to perform I/O; since we're not allowed to backtrack over predicates that do I/O, we have to state that we're content for main to stick to the first solution it finds, whichever one that happens to be.

Now to outline how pick works; here is its definition again:

The first argument is an input, the second and third are outputs. The nondet determinism category means that pick can have any number of solutions, including zero, depending upon its input. Clause 1 says that we can pick an item from a list whose first (head) item is X and whose tail is Xs by returning X as the chosen item and Xs as the remainder. Clause 2 says that we can also pick an item from our list by choosing some Y from Xs, leaving Zs, and returning Y as the chosen item and the list [X | Zs] as the remainder.

For example, The goal pick([1, 2, 3], X, Xs) has three possible solutions:

pick([1, 2, 3], 1, [2, 3])	% By clause 1.
pick([1, 2, 3], 2, [1, 3])	% By clause 2 because
pick([2, 3], 2, [3])	% by clause 1.
pick([1, 2, 3], 3, [1, 2])	<pre>% By clause 2 because</pre>
pick([2, 3], 3, [2])	% by clause 2 because
pick([3], 3, [])	% by clause 1.

Each of these solutions will be enumerated by Mercury on backtracking. (Note that the goal pick([], X, Xs) has no solution because there are no clauses matching the empty list [] as an input.)

Don't worry if much of this seems confusing: once some facility with thinking declaratively (i.e., "what" rather than "how") is acquired, one can look at predicates like **pick** and complicated goals like the condition of this **if-then-else** and immediately understand what is meant. The thing to carry away from this example is the ease with which Mercury allows us to describe a solution to a complex search problem. We have done very little more than write a logical specification of the problem using Mercury syntax, all the tedious operational details are handled for us by the compiler! Elegance of this sort is one of the key things that makes Mercury such an attractive programming language.

Anyway, after all that hard work, let's just prove that all this magic actually works:

```
$ mmc --make crypt
Making Mercury/int3s/crypt.int3
Making Mercury/cs/crypt.c
Making Mercury/os/crypt.o
Making crypt
$ ./crypt
DOG + ANT = CAT
420 + 531 = 951
```

Success!

Points to remember

- Nondeterministic predicates such as **pick** can have multiple solutions for a given set of inputs. Mercury handles this by backtracking to the most recent choice point when a later goal fails.
- As far as possible, try to think declaratively rather than operationally: focus on what it is that is being computed rather than how the computation should proceed. Leave as much of that sort of detail to the compiler as you can.
- Unification goals are quite versatile: they can denote constructions, deconstructions, and equality tests.

Chapter 2

The Mercury type system

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In this chapter we describe the primitive (i.e., built-in) Mercury types, and how to define and use new types. The style of this chapter is a little dry, so we advise the reader to skim through it the first time around, and then dip back into it for reference as occasion demands.

Mercury uses an expressive, statically checked type system similar to that of ML and Haskell. The type system is expressive in the sense that the compiler can always infer the type held in a particular variable; apart from one exception, which is described below, type casts of the kind found in Java and C programs are unnecessary. Static type checking means that any possible type violation in a program is detected at compile-time rather than at run-time. Many common programming errors are, therefore, simply inexpressible in Mercury. A handy secondary benefit of this approach to typing is that the compiler can generate highly optimized code.

Mercury also supports dynamic typing and type reflection for the rare cases where such things are necessary. Dynamic typing is supported by a universal type that can conceal values of any type at all; however, extracting a concealed value does require a checked run-time type cast operation. Type reflection allows a program to examine the structure of values and their types. By way of example, a generic function to compute hash codes for values of any type depends upon type reflection.

Important!

Mercury is a purely declarative language. This means that values are immutable: there is no destructive assignment. Every computation constructs a new value for each output. The compiler may well generate code that uses destructive assignment (e.g., for efficient array updates), but such things are not directly available to the Mercury programmer.

2.1 The primitive types: int, float, string, and char

Note! The underlying representation of the primitive types depends upon the compiler target (C, Java, .Net etc.)

int

Fixed precision integers are represented by the int type. Syntactically, an int is a sequence of digits, optionally preceded by a minus sign (there is no unary plus). The sequence of digits may be decimal, hexadecimal (both uppercase and lowercase are allowed, preceded by 0x), octal (if preceded by 0b).

Examples: decimal -123, 0, 42; hexadecimal -0x7B, 0x0, 0x2a; octal -0o173, 0o0, 0o52; binary -0b1111011, 0b0, 0b101010.

The sequence 0'x denotes the character code for the character x. For example, on an ASCII system 0'a, 0'b, and 0'c denote 97, 98, and 99 respectively.

The int standard library module must be imported to use any of the primitive int operations.

float

Floating point numbers are represented by the float type, which corresponds to the C double type. Syntactically, a float is a decimal floating

point number (the decimal point is required), optionally preceded by a minus sign, optionally followed by an integer exponent.

These are all equivalent: 1.414, 1414e-3, .1414e1, 0.01414e2; either e or E is acceptable as the exponent separator.

The float standard library module must be imported to use any of the primitive float operations. Constants such as pi and e and more complex floating point operations, such as the trigonometric functions, are defined in the math standard library module.

string

A string constant is a sequence of characters enclosed in double quotes.

Examples: " ", "Hello, World!\n", "\'`Lawks!\'' I declared.".

Certain characters have special syntax referred known as *character escapes*:

\"	double quote	$\backslash \backslash$	backslash
\،	single quote	\a	alert ("beep")
∖b	backspace	\r	carriage return
\f	form-feed	\t	tab
\n	newline	\v	vertical tab

Characters can also be specified by character code using the sequence $\ h,$ where hh is a hexadecimal number, or ooo, where ooo is an octal number. the ASCII character A, for example, can also be written as x41 or 101.

Note! For arcane reasons, it is a Very Bad Idea to include the NUL character, \x00\, in strings.

```
A backslash at the end of a line is ignored in string constants. Thus

'`abc\

def"

is equivalent to just "abcdef". Otherwise, literal newlines may appear in a

string constant:

'`pqr

xyz''

is equivalent to "pqr\nxyz".
```

The string standard library module must be imported to use any of the

primitive string operations. In particular it defines ++, the string concatenatation function — for example, "foo" ++ "bar" = "foobar".

char

Character constants are represented using the **char** type and, syntactically, are single characters (or character escapes or character codes as described above) enclosed in single quotes. Characters that could be interpreted as infix operators, such as + and *, should be further enclosed in parentheses.

Examples: 'A', '\x41\', '\101\', '\'', '\n', ('+'), ('*').

XXX Are we ASCII specific?

The char standard library module must be imported to use any of the primitive operations on chars.

2.2 Tuples

A tuple is a fixed size vector of values. Syntactically, a tuple *type* is a comma separated sequence of type names enclosed in braces, whereas a tuple *value* is a comma separated sequence of values enclosed in braces.

Examples: {111, 'b'} is a value of type {int, char}; {1.2, 3, "456"} is a value of type {float, int, string}; {"a", {"little", "contrived"}} is a value of type {string, {string}}.

Note! Unlike lists (described below), tuples are constructed and deconstructed as complete entities. There is no such thing as the head or tail of a tuple. **XXX** Should we include this note at all? Or move the whole tuples section after lists?

Tuples are occasionally useful for aggregating small numbers of different types. More often than not it is better style to use a discriminated union type.

2.3. LISTS

2.3 Lists

A list is a linear sequence of values sharing the same type. Syntactically, a list is a comma separated sequence of values enclosed in brackets.

Examples: [] denotes the empty list, regardless of the list type; [1, 2, 3] is a value of type list(int); ['a', 'b', 'c', 'd'] is a value of type list(char); [[1], [2, 3], [4]] is a value of type list(list(int)).

It is an error to mix values of different types inside a list because there is no way to give a type to values such as [1, "2", 3.4].

The list standard library module defines the list type and a wealth of list operations. If you forget to import the list module then the compiler will issue warnings like this:

foo.m:031: error: undefined symbol '[|]/2' foo.m:031: (the module 'list' has not been imported).

Lists are actually composed of two kinds of building block: [] (the empty list) and []] (the non-empty list constructor, pronounced "cons"). If X is an int, say, and Xs a list(int), then [](X, Xs) is a list(int) whose *head* (first member) is X and whose *tail* (list of trailing members) is Xs.

Lists are so common that special syntactic sugar exists for them: [X | Xs] is shorthand for [] (X, Xs); [X, Y, Z | Ws] is shorthand for [X | [Y | [Z | Ws]]]; and [X, Y, Z] is shorthand for [X, Y, Z | []].

2.4 Discriminated union types

Discriminated unions allow the definition of new, structured types. This example shows a representation of playing cards using discriminated union types:

:-	type	playing_card	>	card(rar	nk, suit))	; joker.				
:-	type	rank	>	ace	; two	;	three	;	four		
			;	five	; six	;	seven	;	eight		
			;	nine	; ten	;	jack	;	queen	;	king.
:-	type	suit	>	clubs ;	diamond	S	; hearts	; :	spades.		

The *data constructors* defining the values of the discriminated union types appear to the right of the arrows: suit has four possible values, rank thirteen, and playing_card fifty three (fifty two possible card values from card(ace, clubs), card(two, clubs), all the way up to card(king, spades), plus the joker option).

An exhaustive switch on a discriminated union type must test for every possible top-level data constructor. That is, an exhaustive switch on a playing_card value need only test for card(_, _) and joker rather than every specific card instance.

(The term "discriminated union" is used because a type denotes a union of sets of possible values, each of which is distinguished by its data constructor.)

Data constructors with named fields

The fields of a data constructor can be named:

:- type bank_account> account(name	::	string	,
	account_no	::	int,	
	funds	::	float).

We can use field names to access fields directly without having to first deconstruct a bank_account value. That is, rather than writing

we can write

```
( if BankAcct^funds >= RequestedSum then
    ... debit RequestedSum from BankAcct ...
else
    ... reject debit request ...
)
```

The Mercury compiler automatically generates a *field access function* for every named field in a data constructor. The **bank_account** field access functions would be defined like this:

account(A, _, _)^name = A. account(_, B, _)^account_no = B. account(_, _, C)^funds = C.

Field access functions to "update" named fields are also generated:

```
( if BankAcct^funds >= RequestedSum then
    NewBankAcct = (BankAcct^funds := BankAcct^funds - RequestedSum)
    else
        ... reject debit request ...
)
```

The expression (BankAcct^{funds} := X) returns a value identical to BankAcct except the funds field will contain X. The bank_account field access update functions would be defined like this:

```
( account(_, B, C)^name := A ) = account(A, B, C).
( account(A, _, C)^account_no := B ) = account(A, B, C).
( account(A, B, _)^funds := C ) = account(A, B, C).
```

Note that you do not have to name every field of a data constructor; unnamed fields can only be "read" or "updated" by explicitly deconstructing or constructing the entire data constructor value.

You are not allowed to use the same field name in different types defined in the same module. This is an error:

:- type cat ---> cat(name :: string). :- type dog ---> dog(name :: string).

Instead use distinct field names, such as cat_name and dog_name, or use a single type with two data constructors (the same field name can be used in different data constructors of the same type.)

A field access may fail if a data type has more than one data constructor. For example, given :- type playing_card ---> card(card_rank :: rank, card_suit :: suit) ; joker.

A goal featuring the expression Card^{card}rank will fail if Card happens to be a joker.

Field accesses can be chained together.

```
:- type employee ---> employee(id :: int, contact :: contact_details).
:- type contact_details ---> contact_details(address :: string, phone :: int).
```

If Employee contains a value of type employee then the expression Employee contact address is the address field of the contact field of the employee constructor.

Nested fields can be updated. The expression Employee contact address := NewAddr denotes a copy of Employee with the address field of the contact field of the employee data constructor updated to hold NewAddr.

Parentheses can change the meaning of an update expression: (Employee^contact)^address := NewAddr denotes an updated copy of the contact field of the employee data constructor (i.e., the type of this expression is contact_details, not employee.)

One final remark: it is also possible to explicitly define field access functions, for instance for "virtual fields" that are computed rather than stored in a data constructor or for update functions that perform sanity checks on their arguments. User defined field access functions are described fully in Chapter **XXX**.

2.5 Polymorphic types

Polymorphic types are types parameterised by *type variables*. A polymorphic binary tree type carrying values at the branches could be defined like this:

:- type tree(T) ---> leaf ; branch(tree(T), T, tree(T)).

This is just a discriminated union type with a type argument, T. T can match any type at all, so tree(int), tree(string), tree(list(char)) are all refinements of tree(T).

Examples: branch(branch(leaf, 1, leaf), 2, branch(leaf, 3, leaf)) is a value of type tree(int); branch(leaf, {'a', 65}, branch(leaf, {'b', 66}, leaf)) is a value of type tree({char, int}); and leaf is a value of every tree type.

The canonical example of a polymorphic data type is the list type defined in the list standard library module:

:- type list(T) ---> [] ; [T | list(T)].

The maybe type defined in the std_util standard library module is another useful polymorphic type:

:- type maybe(T) ---> no ; yes(T).

This type is commonly used to represent optional values (had the database community known about maybe types they never would have invented NULLS and wrecked the relational model...)

Once one has polymorphic types, it is natural to want to define polymorphic predicates and functions. The list module length function works for lists of every type thanks to the type argument in its signature (it is just convention that we have reused the name T here — any variable name would do):

:- func length(list(T)) = int.
length([]) = 0.
length([_ | Xs]) = 1 + length(Xs).

The first clause defines the length of the empty list to be 0; the second clause defines the length of a non-empty list $[_ | Xs]$ to be 1 for the head (the underscore says we don't care what that happens to be) plus the length of the tail, Xs.

Here is an example of a polymorphic predicate to decide whether a given value resides in an ordered binary tree of the type we defined at the start of this section (by ordered we mean that smaller values appear to the left of larger values in the tree):

The ordering function is built-in to Mercury and compares any two values of the same type, returning a result of type comparison_result:

```
:- type comparison_result ---> (<) ; (=) ; (>).
```

As you can see, data constructor names don't necessarily have to be alphanumeric. These data constructor names must appear in parentheses to stop the Mercury parser from interpreting them as infix operators. This is also an example of *overloading*, where the same name may be used for more than one purpose provided there is no ambiguity.

After calling ordering, search switches on O to decide what to do next: if the value at the current branch, X, is less than the value we are searching for, Y, then search should proceed down the right subtree, R. If X = Y then the search terminates successfully. Otherwise, X is greater than Y and the search should proceed down the left subtree, L. The absence of a clause for leaf values means that any search that reaches a leaf will fail.

2.6 Equivalence types

Readability is often improved by giving simple names to complex types or by using more meaningful names for a specific uses of general types:

```
:- type height == float. % In metres.
:- type radius == float. % In metres.
:- type volume == float. % In cubic metres.
:- func volume_of_cylinder(height, radius) = volume.
:- func volume_of_sphere(radius) = volume.
```

XXX Move this stuff about comments to chapter 1. (The % sign introduces a comment, which extends to the end of the line.) Here we define the types height, radius and volume to be equivalent to (i.e., interchangeable with) type float. We could have just declared volume_of_cylinder using

```
:- func volume_of_cylinder(float, float) = float.
```

but then we would be morally obliged to include a comment explaining which arguments correspond to which measurements.

Equivalence types can also be parameterized. For example:

```
:- type dictionary(Key, Value) == list({Key, Value}).
    % search(Dict, Key, Value) unifies Value if there is an association
    % for Key in Dict, but fails otherwise.
    %
:- pred search(dictionary(Key, Value)::in, Key::in, Value::out) is semidet.
search([{K, V} | Dict], Key, Value) :-
    ( if Key = K then Value = V else search(Dict, Key, Value) ).
    % set(Dict, Key, Value) returns an updated version of Dict
    % associating Key with Value.
    %
:- func set(dictionary(Key, Value), Key, Value) = dictionary(Key, Value).
set(Dict, Key, Value) = [{Key, Value} | Dict].
```

2.7 Abstract types

It is virtually always a Bad Idea to reveal implementation detail to the user of a module. Mercury ensures that predicate and function *definitions* are private to a module because they cannot appear in the *interface* section of a module. Abstract types allow the same kind of information hiding for types. An abstract type is one that is *declared* in the *interface* section of a module, but *defined* in the *implementation* section.

Here's how we would use abstract types if we wanted to export the dictionary type defined above:

```
:- module dictionary.
:- interface.
:- type dictionary(Key, Value).
:- pred search(dictionary(Key, Value)::in, Key::in, Value::out) is semidet.
:- func set(dictionary(Key, Value), Key, Value) = dictionary(Key, Value).
:- implementation.
:- implementation.
:- type dictionary(Key, Value) == list({Key, Value}).
search([{K, V} | Dict], Key, Value) == list({Key, Value}).
search([{K, V} | Dict], Key, Value) :-
    ( if Key = K then Value = V else search(Dict, Key, Value) ).
set(Dict, Key, Value) = [{Key, Value} | Dict].
```

Observe the type declaration in the interface section: it gives the name of the type and its arguments, but nothing else. Further down, in the implementation section, we give a definition for dictionary. At some later point we may reimplement dictionary as an ordered list or binary tree or some other more efficient structure. Such a change would not affect the interface of the dictionary module (because dictionary is an abstract type), so no changes would be required by users of the module.

2.8 Higher order types

Mercury considers functions and predicates to be values just as much as it does ints, strings and lists. Consider the higher order map function (as defined in the list standard library module) which takes a function from T1 values to T2 values, a list of T1 values, and returns a list of T2 values:

```
:- func map(func(T1) = T2, list(T1)) = list(T2).
map(_, []) = [].
map(F, [X | Xs]) = [F(X) | map(F, Xs)].
```

The first clause says that mapping over the empty list returns the empty list. The second clause says that mapping the function F over the non-empty

list [X | Xs] is the list whose head is F(X) (i.e., the result of applying F to X) and whose tail is the result of mapping F over Xs.

The thing to observe here is the argument type func(T1) = T2, which illustrates the syntax for function types.

This next example illustrates the syntax for predicate types (this predicate is also defined in the list standard library module):

```
:- pred filter(pred(T),
                                           list(T),
                                                      list(T),
                                                                list(T)
                                                                         ).
:- mode filter(in(pred(in) is semidet),
                                                                         ) is det.
                                           in.
                                                      out,
                                                                out
filter(_, [], [], []).
filter(P, [X | Xs], Ys, Zs) :-
    filter(P, Xs, Ys0, Zs0),
                                                        Zs0
    ( if
             P(X)
                   then Ys = [X |
                                    YsO],
                                            Zs =
                                     Ys0,
                                            Zs = [X |
                   else
                         Ys =
                                                        Zs0]
    ).
```

the goal filter(P, As, Bs, Cs) unifies Bs with the list of members of As that satisfy P and unifies Cs with the list of members of As that don't. The first clause says that filtering the empty list yields two empty lists. The second clause says that filtering [X | Xs] through the predicate P is the result of filtering Xs through P and adding X to the first result if P(X) (i.e., if P succeeds given X), or adding X to the second result if it doesn't.

The first new thing here is the separation of *type* information from *mode* information in the declarations for filter. Mercury requires a separate **mode** declaration if you do not supply mode and determinism details in the **pred** declaration. The two declarations could be combined thus

However, having separate pred and mode declarations highlights that the *type* of filter's higher order argument is written pred(T). (Separate mode declarations are examined in more detail in Chapter XXX.) XXX Do I need to explain the parameterised in mode?

In general, higher order programming with predicates is more complicated than with functions because of the need to also specify the modes for the higher order arguments.

2.9 univ, the universal type

The univ type provides support for dynamically typed programming. Mercury's type system is so expressive that univ is hardly ever necessary. However, should you require it, here is (a slightly abridged version of) the interface to univ as defined in the univ standard library module:

```
:- type univ.
:- func univ(T) = univ.
:- pred univ_to_type(univ::in, T::out) is semidet.
```

The univ function turns an argument of any type into a univ value (this is an example of overloading a name for a type and a function). The univ_to_type predicate turns a univ value into a value of type T (what T denotes depends upon the context of the call to univ_to_type) if that is the type of value contained in the univ, and fails otherwise.

We said earlier that a value like [1, "2", 3.4] would be rejected because it cannot be given a type. This is true, but we can achieve much the same end by writing [univ(1), univ("2"), univ(3.4)], which does have a type, list(univ).

To illustrate the use of univ_to_type, here is a program to print out univs:

```
:- module print_univs.
:- interface.
:- import_module io.
:- pred main(io::di, io::uo) is det.
:- implementation.
:- import_module list, string, univ.
main(!IO) :-
    print_univ(univ(1), !IO),
    print_univ(univ(2"), !IO),
    print_univ(univ(3.4), !IO),
    print_univ(univ(3.4), !IO),
    print_univ(univ({5, 6, 7}), !IO).
```

:- pred print_univ(univ::in, io::di, io::uo) is det.

```
print_univ(U, !IO) :-
  ( if univ_to_type(U, C) then io.format("a char, %c\n", [c(C)], !IO)
    else if univ_to_type(U, S) then io.format("a string, \'`%s\'`\n", [s(S)], !IO)
    else if univ_to_type(U, I) then io.format("an int, %d\n", [i(I)], !IO)
    else if univ_to_type(U, F) then io.format("a float, %f\n", [f(F)], !IO)
    else io.format("no idea...\n", [], !IO)
    ).
```

Compiling and running this program, we get

```
> mmc --make print_univs
Making Mercury/int3s/print_univs.int3
Making Mercury/cs/print_univs.c
Making Mercury/os/print_univs.o
Making print_univs
> ./print_univs
an int, 1
a string, '2',
a float, 3.400000
no idea...
```

So how does univ_to_type know that C is a char, S a string, and so forth? The answer is the compiler automatically infers these types from context: the argument of a c data constructor in an io.format argument list must be a char; the argument of an s data constructor must be a string; similarly I must be an int and F a float. Information about each result type is supplied to the univ_to_type calls via extra arguments inserted by the Mercury compiler.

Full details of the run-time type information (RTTI) scheme are beyond the scope of this book. The interested reader is referred to the documentation for the std_util module in the Mercury Library Reference Manual and the relevant parts of the Mercury Reference Manual.

2.10 Useful types defined in the Mercury standard library

The Mercury standard library defines many useful types, the most common of which are examined in more detail in Chapter ${\bf X}{\bf X}{\bf X}$. These include

bool for Boolean values, digraph for directed graph processing, list for stacks and sequences, map for dictionaries, queue for first-in first-out (FIFO) queues, pqueue for priority queues, random for random numbers, and set for sets. Many more types are defined in the standard library, but the above suffice for the majority of data structures.

2.11 More advanced types...

Mercury's type system includes two more advanced aspects each with a separate chapter. Chapter **XXX** describes type classes and existentially quantified types which exist to support object oriented programming styles. Chapter **XXX** describes types with user-defined equality and comparison relations; these are so-called *non-canonical* types in which a given semantic value may be represented in more than one way.

XXX I haven't really mentioned type inference, nor have I mentioned explicit type qualification of local vars.

Chapter 3

The Mercury mode system

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Mercury programs are really just logical formulae written down using a particular syntax. The Mercury compiler, however, needs extra information to turn these logical formulae into something the computer can execute. Specifically, it needs to know which arguments of a predicate can be inputs and which outputs. This information is conveyed using *argument modes* and *determinism categories*.

This chapter explains the mode system and how it is used.

3.1 Predicates and procedures

Every predicate must have a declaration specifying which arguments can be inputs and which outputs. Consider the following implementation of a telephone directory in which phone numbers can be looked-up by name:

:- pred phone(string::in, int::out) is semidet.

phone("Ian",	66532).
phone("Julien",	66532).
phone("Peter",	66540).
phone("Ralph",	66532).
phone("Zoltan",	66514).

The style of **pred** declaration used here is called a **pred-mode** declaration and is syntactic sugar for two separate declarations:

:- pred phone(string, int).
:- mode phone(in, out) is semidet.

The plain **pred** declaration tells us the argument types; the **mode** declaration tells us the *argument modes* (in or out) and the corresponding *determinism* category (semidet).

It is natural to want to call some predicates in more than one way — if we want to use **phone** to perform "reverse look-ups", for instance. In such cases more than one **mode** declaration is necessary (**pred-mode** shorthand can only be used for singly-moded predicates).

To allow reverse look-ups with phone all that is necessary is to (a) use separate pred and mode declarations and (b) add an extra mode declaration:

:- pred	phone(string,	int).		
:- mode	phone(in,	out)	is	semidet.
:- mode	phone(out,	in)	is	nondet.

The first mode declaration

:- mode phone(in, out) is semidet.

says that if we call phone giving its first argument as an input and taking its second as an output, then the result is semideterministic: every name in the directory appears exactly once, but not all names are listed. The goal phone("Harald", HaraldsNum) will obviously fail, but phone("Ralph", RalphsNum) will succeed unifying RalphsNum with 66532.

The second mode declaration

:- mode phone(out, in) is nondet.

says that if we call phone with its second argument given as input and take its first argument as output, then the result is nondeterministic: it can fail because the goal phone(Person, 12345) will fail, but the goal

phone(Person, 66532) has no less than three possible solutions — Person = "Ian", Person = "Julien", and Person = "Ralph" — each of which will be computed on backtracking.

Note! Each mode declaration specifies a *procedure* to be derived from the predicate definition. The compiler generates code separately for each procedure of a predicate, reordering goals to ensure that every variable is instantiated (e.g., by some earlier unification or call) before it is used in an input in a unification or call. Mode information is also used to decide whether a unification with a data constructor is a construction or deconstruction.

The compiler verifies that the determinism category for a procedure properly describes the behaviour of the procedure. The compiler will issue an error if a procedure can fail or have multiple solutions when its declared determinism category says otherwise. Moreover, the compiler will report an error if a deterministic switch is incomplete, telling you which cases have been missed (Prolog programmers dream of having error detection like this...)

Implied modes

An *implied mode* is one where an output argument is supplied as an input in a procedure call. Consider the goal phone("Ralph", 66540). The compiler gets around this situation by placing a new, temporary variable in the output position and then adding a unification goal, giving phone("Ralph", Tmp), Tmp = 66540.

3.2 The determinism categories

A determinism category tells us whether a particular procedure can fail and whether it may have more than one solution:

Determinism category	Number of solutions
det	1
semidet	≤ 1
multi	≥ 1
nondet	≥ 0
failure	0

There are three other determinism categories that are only occasionally needed: **erroneous**, which is used for predicates that only terminate by throwing an exception (exceptions are described in Chapter **XXX**), and <code>cc_multi</code> and <code>cc_nondet</code> which are used for committed-choice nondeterminism (see Chapter XXX).

Some examples

```
:- pred square(int::in, int::out) is det.
square(X, X * X).
```

square is det: it cannot fail and every input has a single solution for the output.

```
:- pred absolute_square_root(float::in, float::out) is semidet
absolute_square_root(X, AbsSqrtX) :-
    X >= 0.0,
    AbsSqrtX = math.sqrt(X).
```

absolute_square_root is semidet: it fails for negative inputs while non-negative inputs each have a single solution.

```
:- pred small_prime(int::out) is multi.
small_prime(2).
small_prime(3).
small_prime(5).
small_prime(7).
```

small_prime is multi: it cannot fail and it has more than one solution.

```
:- pred small_prime_factor(int::in, int::out) is nondet.
small_prime_factor(X, P) :-
    small_prime(P),
    X mod P = 0.
```

```
small_prime_factor is nondet: small_prime_factor(11, A), for instance,
will fail, but small_prime_factor(6, A) has solutions A = 2 and A = 3.
```

Finally, the built-in predicate false, which takes no arguments, has determinism failure: it never succeeds. The opposite of false is the built-in predicate true which has no arguments and always succeeds (and is therefore det).

Note! The determinism category of a goal with no output arguments is either det, semidet, or failure. Consider the following:

```
:- pred has_small_prime_factor(int::in) is semidet.
has_small_prime_factor(X) :-
    small_prime(P),
    X mod P = 0.
```

```
Because there are no outputs, Mercury ensures that the goal has_small_prime_factor(15), say, will not succeed more than once, even though small_prime(P) has two solutions, P = 3 and P = 5, satisfying 15 mod P = 0.
```

3.3 Determinism

These rules specify how determinism categories for compound goals are derived (with a little experience this quickly becomes second nature). The determinism category of a goal is derived from the instantiation state of its arguments at the time the goal is executed.

Note! Remember that the compiler reorders the goals in a predicate separately for each mode declaration for the predicate. A running program does not make decisions about which procedures should be executed when calling predicates; this is decided in advance by the Mercury compiler.

Unifications

Whether a unification is a construction, deconstruction, assignment or equality test depends upon which variables are instantiated and which are not at the time the unification is executed.

A unification $X = data_ctor(Y1, Y2, Y3)$ is a *construction* if Y1, Y2, and Y3 are initially instantiated and X is not. Constructions are always det. Afterwards, X will be instantiated.

A unification $X = data_ctor(Y1, Y2, Y3)$ is a *deconstruction* if X is initially instantiated. Afterwards, Y1, Y2, and Y3 will be instantiated. Deconstructions are almost always semidet (in certain circumstances a deconstruction may have determinism category det if it is guaranteed to succeed or failure if it is guaranteed to fail).

A unification X = Y is an *assignment* if precisely one of X or Y is initially instantiated. Afterwards, both variables will be instantiated. Assignments are always det.

A unification X = Y is an *equality test* if both X and Y are initially instantiated. Equality tests are always semidet.

Procedure calls

For a predicate call p(X1, X2, X3), which procedure of p is executed depends upon which mode declaration for predicate p best matches the instantiation states of X1, X2, and X3 at the time the call is executed. The determinism category of the goal is that of the called procedure, adjusted for any implied modes (i.e., extra unifications added because some output arguments of the procedure are already instantiated at the time of the call).

For example, the goal phone("Zoltan", ZoltansNumber) is compiled as a call to the (in, out) is det procedure of phone. The goal phone(Person, 66540) is compiled as a call to the (out, in) is nondet procedure of phone. The goal phone("Ralph", 66532) requires an implied mode and may be compiled either as phone("Ralph", Tmp), Tmp = 66532 or phone(Tmp, 66532), Tmp = "Ralph", both of which are semidet.

Conjunction

A sequence of goals separated by commas, G1, G2, G3, ..., is called a *conjunction*. The commas are pronounced "and" and each subgoal is called a *conjunct*.

A conjunction can fail if any conjunct can fail.

A conjunction can succeed if every conjunct can succeed.

A conjunction can have multiple solutions if it can succeed and one or more

conjuncts have multiple solutions.

Note! These rules are a conservative (i.e., safe) approximation. For example, the
 compiler will conclude that conjunction small_prime(X), X = 4 is semidet,
 even though we can see that this goal has to fail.

Disjunction

A sequence of goals separated by semicolons, (G1; G2; G3; ...), is called a *disjunction*. The semicolons are pronounced "or" and each subgoal is called a *disjunct*.

A disjunction can succeed if any disjunct can succeed.

A disjunction can have multiple solutions if more than one disjunct can succeed or one or more disjuncts can have multiple solutions.

Note! Switches are a special case. A switch is a disjunction that deconstructs a particular variable against a different data constructor in each disjunct. If, apart from the deconstructions, every disjunct is det, then the switch is det if the set of deconstructions is exhaustive and semidet if not.

For example, even though both p and q (below) define switches on X, p is det because its switch is exhaustive, whereas q is semidet because its switch is not:

```
:- type ott ---> one ; two ; three.
:- pred p(ott::in, int::out) is det.
p(X, Y) :- ( X = one, Y = 1 ; X = two, Y = 2 ; X = three, Y = 3 ).
:- pred q(ott::in, int::out) is semidet.
q(X, Y) :- ( X = one, Y = 1 ; X = three, Y = 3 ).
```

- Note! Disjunction binds less tightly than conjunction: (G11, G12, G13; G21; G31, G32) is equivalent to ((G11, G12, G13); G21; (G31, G32)).
- **Note!** A definition spanning multiple clauses is equivalent to a definition using a single clause containing a disjunction. That is

p(one, 1).
p(two, 2).
p(three, 3).

is semantically and operationally identical to

p(X, Y) := (X = one, Y = 1; X = two, Y = 2; X = three, Y = 3).

Note! If any disjunct instantiates a variable that is used outside the disjunction, then every disjunct in the disjunction must also instantiate that variable. That is, the Mercury compiler will report a mode error if a program contains a disjunction that instantiates X in some disjuncts, but not others, and X is also needed outside the disjunction. For instance, the following is illegal because Y, which appears outside the disjunction, is instantiated in the first and second disjuncts, but not the third:

:- pred p(number::in, int::out) is det. p(X, Y) :- (X = one, Y = 1 ; X = two, Y = 2 ; X = three).

Negation

A goal (not G) is called the *negation* of G. The negation fails if G succeeds, and vice versa. The negation succeeds if G fails and fails if G succeeds.

- **Note!** G is said to occur inside a *negated context* and is not allowed to instantiate variables that also occur outside the negation.
- Note! Negation binds more tightly than conjunction, hence not G1, G2, ... is equivalent to (not G1), G2, To negate a conjunction, put the conjunction in parentheses: not (G1, G2, ...)
- **Note!** X = Y is syntactic sugar for not (X = Y).

If-then-else goals

The declarative semantics for a goal (if Gc then Gt else Ge) are identical to those of (Gc, Gt ; (not Gc), Ge). The operational semantics

are more efficient, though: if there are no solutions to Gc, the program immediately executes Ge.

If any of Gc, Gt, or Ge can fail then the if-then-else can fail.

If any of Gc, Gt, or Ge can have multiple solutions then the if-then-else can have multiple solutions.

- **Note!** Gc is not allowed to instantiate variables that are used outside the if-thenelse. This is because, semantically, Gc appears in a negated context. It is all right, however, for Gc to instantiate variables that are used by Gt.
- Note! Execution can backtrack into Gc. For example,

(if small_prime(X), X > 2 then Y = X * X else Y = -1)

has solutions Y = 9, Y = 25, and Y = 49.

Prolog programmers take note: unlike Mercury, Prolog programs commit to the first solution of Gc. The Prolog equivalent of the above goal would have Y = 9 as its only solution, not Y = 25 or Y = 49.

Note! (Gc -> Gt ; Ge) is an alternative, albeit old-fashioned, syntax for (if Gc then Gt else Ge).

3.4 Procedures and code reordering

The aim of this section is to give the reader some understanding of code reordering. This knowledge is not required to write Mercury programs, but it can help the programmer understand mode-related error messages from the compiler.

We will illustrate using the append predicate defined in the list standard library module. The declarative semantics of append(Xs, Ys, Zs) is that the list Zs is the concatentation of lists Xs and Ys. So append([1], [2, 3], [1, 2, 3]) is true, but append([2, 3], [1], [1, 2, 3]) is not.

:- pred append(list(T), list(T), list(T)).
:- mode append(in, in, out) is det.
:- mode append(out, out, in) is multi.
append(Xs, Ys, Zs) :-

(
 Xs = [], Zs = Ys
;
 Xs = [X | Xs0], append(Xs0, Ys, Zs0), Zs = [X | Zs0]
).

This code needs no reordering for the (in, in, out) is det mode, where Xs and Ys start off instantiated and Zs starts off uninstantiated. The first disjunct works like this:

1	Xs = []	Deconstruct Xs
2	Zs = Ys	Assign Zs

and the second disjunct works like this:

1	Xs = [X Xs0]	Deconstruct Xs, instantiating X and Xs0
2	append(Xs0, Ys, Zs0)	Call the (in, in, out) procedure, instanti-
		ating Zs0
3	Zs = [X Zs0]	Construct Zs

Because each disjunct deconstructs Xs in a different way, this disjunction is a switch. Because the switch is exhaustive, and the other goals in each disjunct are det, the switch as a whole is det.

The (out, out, in) is multi mode, where only Zs is initially instantiated, does require some reordering in order to ensure that every variable is instantiated before it is needed. The first disjunct becomes

1	Xs = []	Construct Xs
2	Zs = Ys	Assign Ys

and the second disjunct becomes

1	Zs = [X Zs0]	Deconstruct Zs, instantiating X and Zs0
2	append(Xs0, Ys, Zs0)	Call the (out, out, in) procedure, instan-
		tiating XsO and Ys
3	Xs = [X Xs0]	Construct Xs

Since this disjunction is not a switch and the first disjunct always leads to a solution, the disjunction as a whole is multi in this case.

Note! The Mercury compiler reorders code as little as possible. However, programmers should not write code that depends upon any particular order of evaluation — code can also be reordered by various optimizations! In particular, it

is a bad idea to write something like (if $X \ge 0$, Z = Y / X then ... else ...), assuming that the test for X being non-zero will guarantee that this code cannot lead to a division-by-zero error at run-time. It is certainly possible that the test and the division may be compiled the other way around.

3.5 Insts and subtypes

So far we have only talked about variables going from being uninstantiated to being instantiated. It turns out to be useful to also keep track of the possible values a variable can have when it is instantiated. Mercury uses insts for this purpose. An inst represents the possible instantiation states of a variable at a particular point in a program.

The most basic **inst**s are **free**, meaning a variable is uninstantiated, and **ground**, meaning a variable is instantiated with some unknown value of the appropriate type.

The built-in modes in and out are defined using the following syntax:

:- mode in == (ground >> ground).
:- mode out == (free >> ground).

That is, an **in** mode argument of a goal must be **ground** (i.e., be instantiated with some value) before the goal is executed and will also be **ground** afterwards, while an **out** mode argument must be **free** (i.e., not instantiated) before the goal is executed, but will be **ground** afterwards.

Note! If a goal fails or backtracks then the **inst**s of its arguments stay the same as they were before the goal was tried.

Specialised insts

It is occasionally useful to define new insts matching only subsets of possible values that a variable might have. Consider the inst non_empty_list which is defined in the list standard library module:

:- inst non_empty_list == bound([ground | ground]).

```
:- pred head(list(T), T).
:- mode head(in, out) is semidet.
:- mode head(in(non_empty_list), out) is det.
head(Xs, X) :- Xs = [X | _].
```

The inst declaration defines non_empty_list to mean "bound to the list data constructor []] whose first argument has inst ground and whose second argument has inst ground".

The first mode for head tells us that if all we know about the first argument is that it is ground (i.e., it could be bound to any value of type list(T), including []) then a call to head is semidet.

The second mode for head says that if we know the first argument is a nonempty list (i.e., whatever value it has, it's top-most data constructor must be [1] with two ground arguments) then a call to head is guaranteed to succeed.

The second mode declaration uses the built-in parameterised form of the in argument mode, which is defined like this:

:- mode in(I) == (I >> I).

where I is an inst parameter. in(non_empty_list) is therefore equivalent to writing (non_empty_list >> non_empty_list). There is also a built-in parameterised out argument mode, defined thus:

```
:- mode out(I) == (free >> I).
```

When compiling the procedure for the second mode of head, the Mercury compiler uses the information about the inst of the first argument, Xs, to infer that the goal $Xs = [X | _] must (a)$ be a deconstruction and (b) must succeed because whatever value Xs has matches the pattern $[_ | _]$.

Note! A value with a bound(...) inst can always be used in a context where a ground value is expected, but not the other way around.

XXX Mention the alternative inst definition syntax.

3.6. UNIQUENESS

Recursively defined insts

It is possible to describe quite complicated instantiation states. The following insts, for instance, describe lists of even and odd lengths respectively:

```
:- inst even_length_list == bound([]; [ground | odd_length_list]).
:- inst odd_length_list == bound([ground | even_length_list]).
```

The first inst declaration defines even_length_list to mean "bound *either* to [] *or* to [|] with two arguments, the first having inst ground and the second having inst odd_length_list (multiple possibilities in a bound expression are separated by semicolons).

The second inst declaration defines odd_length_list to mean "bound to [1] with two arguments, the first having inst ground and the second having inst even_length_list."

Partial instantiation

A partially instantiated value is one whose inst is bound(...) where the ... part contains free sub-insts, either directly or indirectly.

Partial instantiation is not currently supported for several reasons, including the difficulty of analysing such code, the difficulty of maintaining such code, and the difficulty of compiling such code efficiently.

3.6 Uniqueness

A bound(...) inst is said to be *shared* — that is, it corresponds to a value that may be referred to, directly or indirectly, by more than one variable at a given point in the program.

Mercury has a special inst, unique, which is like ground, but it means that there is precisely one reference to the unique data at this point in the program. The counterpart to unique, is clobbered. A variable with inst clobbered may never be used again (e.g., because the value it refers to is now out-of-date or has been overwritten with something else). The most common use of uniqueness is for managing IO. All the IO operations defined in the io standard library module include two arguments of type io, with modes di and uo respectively. di stands for "destructive input" and uo stands for "unique output". These modes are built-in and defined thus:

```
:- mode di == (unique >> clobbered).
:- mode uo == (free >> unique).
```

To illustrate, consider these pred declarations taken from the io module:

```
:- pred io.write_string(string::in, io::di, io::uo) is det.
:- pred io.write_int(int::in, io::di, io::uo) is det.
:- pred io.nl(io::di, io::uo) is det.
```

and the following code snippet:

```
io.write_string("The meaning of life is ", IOO, IO1),
io.write_int(42, IO1, IO2),
io.nl(IO2, IO3)
```

The io type arguments denote "states of the world". These io states are updated when IO actions are performed. One can never go back to an earlier state (you can't unplay a piece of music or unprint a document), so each IO action clobbers the io state passed to it and produces and new io state as its result. Similarly, because one cannot copy the state of the world, io states have to be unique. These constraints ensure that the above code snippet executes in the expected order — that is, first the string 'The meaning of life is '' will be printed (clobbering IO0 and producing IO1), then the number 42 (clobbering IO1 and producing IO2), and finally a newline (clobbering IO2 and producing IO3).

Say we were to accidentally reuse IOO in the second goal:

```
io.write_string("The meaning of life is ", IOO, IO1),
io.write_int(42, IOO, IO2),
io.nl(IO2, IO3)
```

The Mercury compiler will report the following error (line 27 in file foo.m is the call to io.write_string):

foo.m:027: In clause for 'main(di, uo)':
foo.m:027: in argument 2 of call to predicate 'io.write_string/3':
foo.m:027: unique-mode error: the called procedure would clobber
foo.m:027: its argument, but variable 'IOO' is still live.

Note! Procedures that can clobber arguments must have determinism category det or cc_multi. They must always succeed and produce a single result. The reason for this is that once an argument is clobbered, which could happen at any point during the execution of the procedure, there is no way of unclobbering it on failure or backtracking. Consequently it is also an error for code to backtrack into such procedures. For the rare cases where one needs to do such things, the reader is referred to the section on backtrackable destructive update in the Mercury reference manual which discusses "mostly uniqueness".

Other modules in the standard library that use uniqueness are **array** and **store**. The **array** module implements arrays with O(1) look-up and set operations. It achieves this by using destructive update for the array set operation. This is quite safe because arrays are unique: the 'old' version of the array is clobbered by the update operation so it can never be referred to again; the 'new' version of the array is simply the updated 'old' version. The **store** module allows one to construct safe, pointer-based structures. Pointer referents can be accessed and updated in O(1) time by using the same technique as for arrays. Chapter XXX describes these modules in more detail.

3.7 Higher-order modes

XXX Fill this out.

3.8 Committed-choice nondeterminism

XXX Fill this out.

The cc_nondet and cc_multi modes. There may be multiple solutions to a cc predicate, but you will only get one of them.

The compiler will report an error if a program can backtrack into a committedchoice goal: all goals following a committed-choice goal must be *guaranteed* to succeed. Programming under this restriction is quite burdensome.

If all solutions to a committed-choice predicate are equivalent, in the sense that, no matter which solution you get, the observable behaviour of your program will be the same, then you can use the built-in function promise_only_solution to escape from the committed-choice context.